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Executive Summary

Slowly Time-Dependent Ratio between Recovery and Infection Rates: SIR-Solution

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The susceptible-infectious-recovered/removed (SIR) model was first applied to the mathematical theory of epidemics about a century ago. It illustrates the basic compartment model, which divides any population of N individuals into the three fractions of susceptible (S), infected (I), and recovered/removed (R) individuals, along with later enhancements. The transitions from the susceptible to the infected compartment and from the infected to the recovered/removed compartment are controlled, respectively, by the infection rate, $\alpha(t)$, and the recovery/removed rate, (t). The current study examines analytical SIR equation solutions for various time-dependent infections and recovery rates such that their ratio is no longer constant and starts to vary with time as well.

Except for, which investigated exceptional analytical solutions based on non-constant ratios, $k(S)$, being of the polynomial form of order S4 or less in the case of positive or negative powers, this general case has not, as far as we are aware, been examined in the literature. The latter makes use of the vastly differing time scales of motion of the light and heavy particles in the system to simplify the solution of the quantum mechanical equations of motion. One must deduce the real-time dependencies of the infection and recovery rates from the adopted parameterized reduced time dependence of their ratio to derive the real-time dependence from the reduced time dependence of the SIR quantities of interest.

The transitions from the susceptible to the infected compartment and from the infected to the recovered compartment are controlled by these two rates, respectively. Numerous numerical solutions of the SIR equations, starting with the ground-breaking investigations, have assumed stationary values for the two rates, which was also the case in the analytical solution. In the current study, estimated SIR solutions are generated for various time dependencies of the infection and recovery rates so that their ratio no longer remains constant and becomes likewise time-dependent. This is reportedly the first time this has been done. The analysis here is based on the adiabatic approximation, which considers time-dependent ratios $k(t)$, which slowly vary in comparison to the typical pandemic wave time characteristics.

The analytical advancements that are now accessible will undoubtedly make it easier to manage aggressive pandemic breakouts in the future. The improvements include both the adiabatic approximation for the slowly time-dependent ratios of the infection and recovery rate presented here and the fairly accurate explicit analytical solutions of the SIR equations and its generalizations, such as the SIRV-equations (which includes vaccination denoted by "V"). The improvements should enable a

more accurate prediction of the temporal evolution of pandemic outbursts, taking non-pharmaceutical interventions into account. This will enable predictions of the tolerable maximum seven-day incidence value for given health capacities in various countries, as well as the estimate of fatality rates and the total number of fatalities.

However, far better and more thorough monitoring of the rate of newly infected individuals is required for such future quantitative research.

KEYWORDS
Epidemiology; statistical analysis; time-scale separation; differential equations; adiabatic approximation